Generation of Equivalent Circuit from Finite Element Model of Electromagnetic Devices Using Proper Orthogonal Decomposition

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This paper presents generation of equivalent circuits from finite element (FE) model of electromagnetic devices using proper orthogonal decomposition (POD). This method effectively computes the frequency response of the reduced FE model which is constructed by POD-based model order reduction. Then the lumped parameters for the equivalent circuit are determined so as to minimize the error between the frequency responses of the reduced FE model and circuit. The frequency characteristics of a 3D inductor evaluated by the equivalent circuit are shown to be in good agreement with those computed from the original FE model.

***Index Terms*— Equivalent circuits, finite element analysis, reduced order systems.**

I. Introduction

FINITE element (FE) analysis has widely been performed for design of electric machines and devices. However, due to its long computational time, equivalent circuits are frequently used rather than FE analysis for design of driving and control circuits. To obtain equivalent circuits for motors, for example, lumped parameters are determined from loss and inductances computed by FE analysis [1]. It would be, however, difficult to obtain accurate frequency characteristics over a wide range using this conventional method.

Recently the equivalent circuit of ladder configuration has been directly generated from the analytical expression of an eddy current problem for thin iron sheets [2]. It is shown that the discrepancy between the frequency characteristics of the generated equivalent circuit and the analytical solution can be reduced by increasing the number of ladder stages. Now a question arises, is it possible to generate the equivalent circuit not from the analytical solutions which can be obtained only to simple problems but from FE models? The equivalent circuits of FE models could be generated if its frequency characteristics are available. However, one needs long computational time to obtain frequency characteristics of large FE models.

In this paper, we propose a generation method of equivalent circuit of FE models using model order reduction (MOR) base ond proper orthogonal decomposition (POD) [3]. In this method, the computational time necessary for FE analysis can be reduced by POD-based MOR. The frequency response of the FE models is thus effectively computed for the circuit generation. The lumped parameters in the circuit are determined so as to minimize the error between the frequency responses of the reduced FE model and circuit.

We apply the present method to three dimensional inductor model and compare the accuracy and computational time of the resultant equivalent circuit with those of the original FE model.

II. Proper orthogonal decomposition

Let us consider the FE equation in the frequency-domain,

$$\mathrm{K}(\mathrm{j}\omega)\boldsymbol{x} = \boldsymbol{b}(\mathrm{j}\omega) \tag{1}$$

where $\mathrm{K}\in\mathrm{R}^{n\times n}$, $\boldsymbol{x}\in\mathrm{R}^n$, $\boldsymbol{b}\in\mathrm{R}^n$ and ω are FE matrix, solution vector, source vector and angular frequency, respectively. We solve (1) at s snapshot points to construct the data matrix X

$$\mathrm{X} = \begin{bmatrix}\boldsymbol{x}_1(\omega_1) & \boldsymbol{x}_2(\omega_2) & \cdots & \boldsymbol{x}_s(\omega_s)\end{bmatrix} \tag{2}$$

The singular value decomposition applied to X results in

$$\mathrm{X} = \mathrm{W}\Sigma\mathrm{V}^{\mathrm{t}} = \sigma_1\boldsymbol{w}_1\boldsymbol{v}_1^{\mathrm{t}} + \sigma_2\boldsymbol{w}_2\boldsymbol{v}_2^{\mathrm{t}} + \cdots + \sigma_s\boldsymbol{w}_s\boldsymbol{v}_s^{\mathrm{t}} \tag{3}$$

where σ_i is ith eigenvalue of X, $1 \le i \le s$, and $\boldsymbol{w}_i$, $\boldsymbol{v}_i$ are the eigenvectors of XX^{t} and $\mathrm{X}^{\mathrm{t}}\mathrm{X}$, respectively. The solution $\boldsymbol{x}$ can be approximately expressed by the linear combination of the eigenvectors, that is, $\boldsymbol{x}=\mathrm{W}\boldsymbol{y}$ where $\boldsymbol{y}\in\mathrm{R}^s$. Thus (1) becomes

$$\mathrm{W}^{\mathrm{t}}\mathrm{K}(\mathrm{j}\omega)\mathrm{W}\boldsymbol{y} = \mathrm{W}^{\mathrm{t}}\boldsymbol{b}(\mathrm{j}\omega) \tag{4}$$

The snapshot number s is set much smaller than n so that one solves (4) much faster than (1) at the sampling points to obtain the frequency response.

III. Generation of equivalent circuit

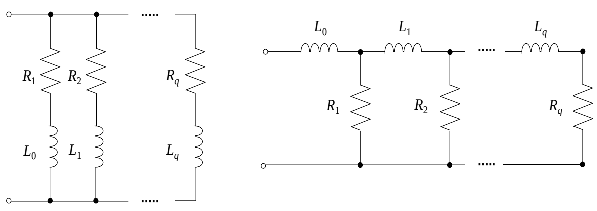
Next, we generate the equivalent circuit from the frequency response computed by POD-based MOR. We employ here Foster and Cauer circuits shown in Fig. 1 [4]. In Foster circuit, the admittance $Y(\mathrm{j}\omega)$ is expressed by

$$Y(\mathrm{j}\omega) \approx \sum_{k=0}^{q}\frac{1}{R_k + \mathrm{j}\omega L_k} \tag{5}$$

where R_k, L_k and q are resistance, inductance and number of the stage of the ladder circuit, respectively. In the Cauer circuit, the impedance $Z(\mathrm{j}\omega)$ is expressed in a form of continued fraction as

$$Z(\mathrm{j}\omega) \approx \mathrm{j}\omega L_0 + \frac{1}{R_0 + 1/\left(\mathrm{j}\omega L_1 + 1/\left(R_1 + \cdots\right)\right)} \tag{6}$$

To determine the lumped parameters $R_1,\cdots, R_q$ and $L_1,\cdots, L_q$ in the equivalent circuit, we introduce the optimization problem defined by



(a) Forster circuit (b) Cauer circuit

Fig. 1. Equivalent circuits

$$f(\boldsymbol{R},\boldsymbol{L}) = \sqrt{\sum_{i}^{M} \left| G^{\mathrm{FEM}}(\mathrm{j}\omega_i) - G(\mathrm{j}\omega_i, \boldsymbol{R}, \boldsymbol{L}) \right|^2} \rightarrow \min. \quad (7)$$

subject to $R_k, L_k \geq 0$

where $\boldsymbol{R} = [R_1, R_2, \cdots, R_q]$, $\boldsymbol{L} = [L_1, L_2, \cdots, L_q]$, $G^{\mathrm{FEM}}(\mathrm{j}\omega_i)$, $G(\mathrm{j}\omega_i,\boldsymbol{R},\boldsymbol{L})$ are obtained from reduced equation (4) and its equivalent circuit and M is the number of sampling points. The optimization problem (7) is solved here by the real coded genetic algorithm (RGA).

IV. Numerical Results

We consider the three dimensional inductor model shown in Fig. 2 connected to the simple circuit shown in Fig. 3 where electrical conductivity κ, relative permeability μ_r, R and L are set to 5×10^6 S/m, 10, 10^{-5} Ω and 10^{-15} H. The numbers of snapshots s and frequency sampling points M are set to 3 and 11, respectively.

The frequency characteristics of the current obtained from the original FE model and the present method for different number of ladder stages q are shown in Figs. 4 and 5. In both circuits, the results of the present method are in good agreement with those of FE model when $q\geq3$. The computational time for generation of the both circuits is found to be about 25% of that generated from original FE model.

TABLE I shows the resultant lumped parameters in both circuits when q=5.

TABLE I

Circuit Parameter

Foster circuit				
R_0[Ω]	R_1[Ω]	R_2[Ω]	R_3[Ω]	R_4[Ω]
1.43×10^{-4}	1.64×10^{-4}	3.31×10^{-4}	1.22×10^{-5}	5.19×10^{-4}
L_0[H]	L_1[H]	L_2[H]	L_3[H]	L_4[H]
6.25×10^{-7}	7.16×10^{-8}	1.78×10^{-8}	3.71×10^{-8}	2.18×10^{-6}
Cauer circuit				
R_0[Ω]	R_1[Ω]	R_2[Ω]	R_3[Ω]	R_4[Ω]
5.15×10^{-5}	3.46×10^{-5}	2.38×10^{-5}	1.03×10^{-4}	4.07×10^{-3}
L_0[H]	L_1[H]	L_2[H]	L_3[H]	L_4[H]
1.41×10^{-8}	2.08×10^{-8}	7.07×10^{-12}	2.00×10^{-7}	3.56×10^{-9}

V. Conclusion

In this paper, we have proposed a generation method of equivalent circuits from reduced FE models. We have applied the present method to three dimensional inductor model. The proposed method has been shown sufficiently accurate for this example when $q\geq3$. This present method can be applied not only to Foster and Cauer circuits, but also more complex circuits, such as, lumped element equivalent circuit [5] for antennas, which will be discussed in the full paper.

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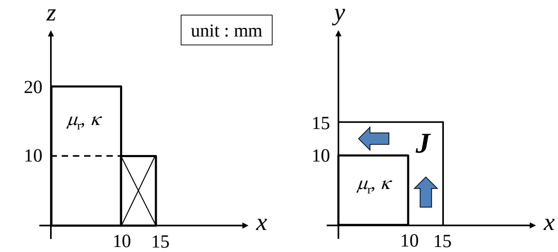

Fig. 2. Inductor model

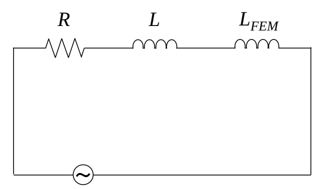

Fig. 3. Circuit including finite element model

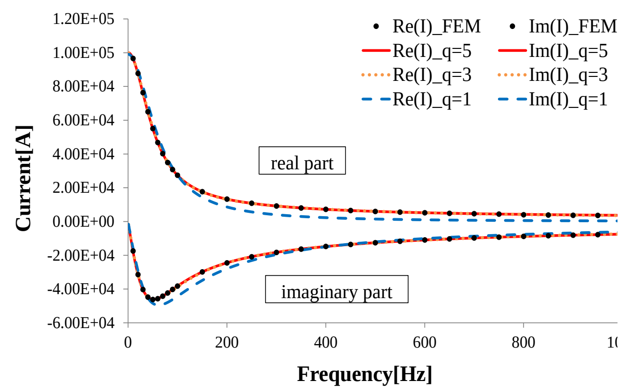

Fig. 4. Current-Frequency (Foster)

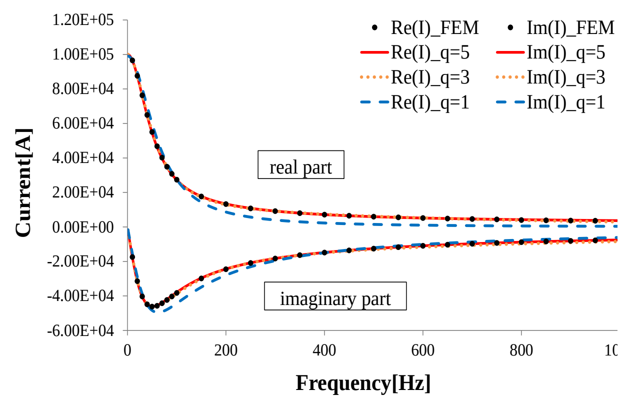

Fig. 5. Current-Frequency (Cauer)

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